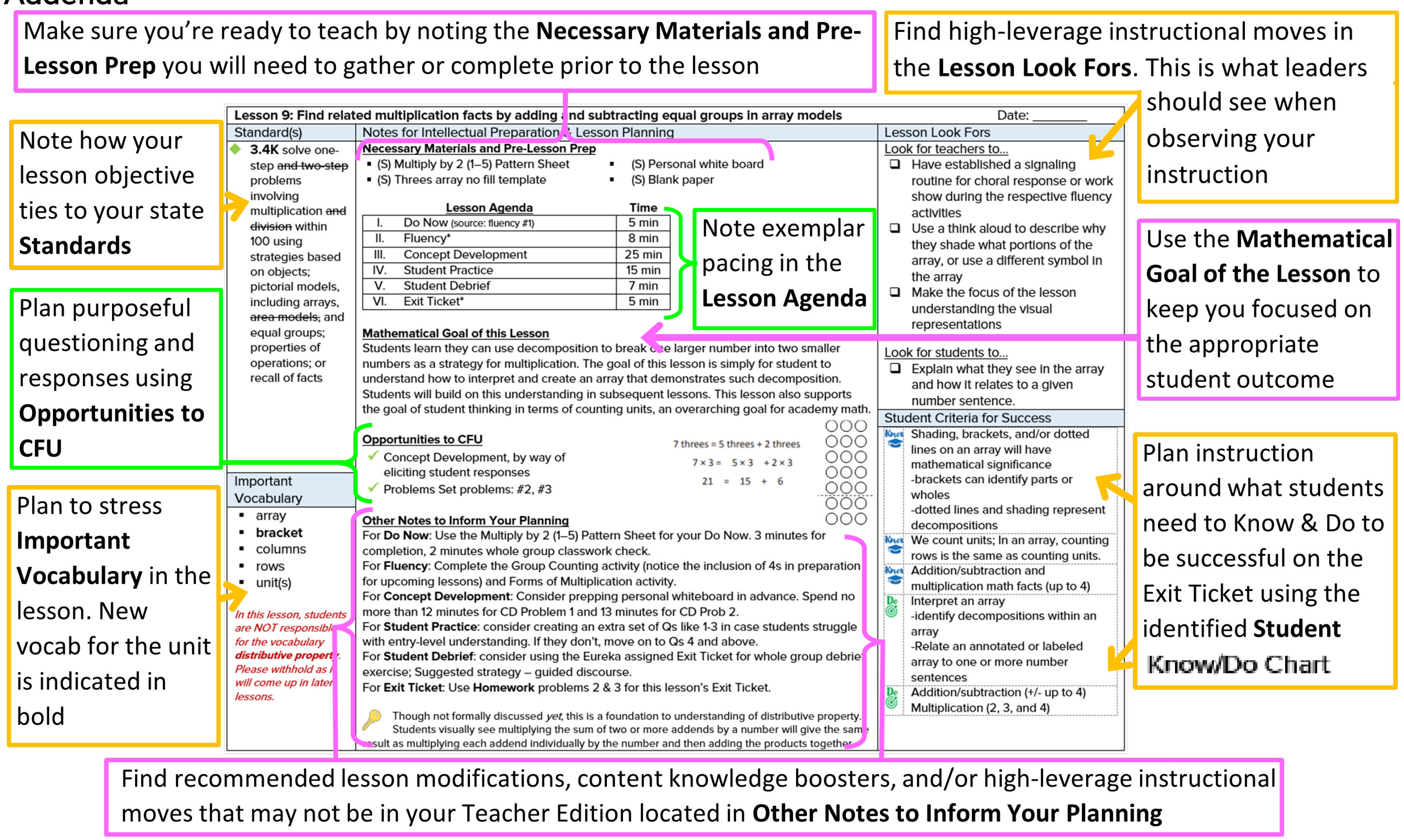
Table of Contents

How to Use This Addenda



UNIT SYNOPSIS
Vectors are useful tools for modeling and solving real-life problems involving magnitude and direction. Quantities such as force and velocity involve both magnitude and direction and cannot be completely characterized by a single real number. In this unit, we will explore the fundamentals of vectors such as vector notation, vector addition, vector scalar multiplication, and the dot product. Later in the unit, we will connect basic trigonometry when calculating the angle between two vectors using the dot product. Mid-unit covers parametric equations and their graphs. Parametric equations help us model motion of an object and is defined by at least two continuous functions of time, t. We will explore and discuss their graphs as well as how it's used to model projectiles in motion. In the previous units, we plotted and analyzed functional characteristics of graphs on the Cartesian plane. The last part of this unit, we introduce a different way of locating points and graphing curves using polar coordinates.

CONTENT STANDARDS

Below are the standards addressed in this unit.

Texas Essential Knowledge and Skills (TEKS)	
Knowledge and Skills	**Student Expectations (SE)**
(3) Relations and Geometric Reasoning The student uses the process standards in mathematics to **model** and make **connections** between **algebraic** and **geometric relations**.	**(3.A)** Graph a set of parametric equations. **(3.B)** Convert parametric equations into rectangular relations and convert rectangular relations into parametric equations. **(3.C)** Use parametric equations to model and solve mathematical and real-world problems. **(3.D)** Graph points in the polar coordinate system and convert between rectangular coordinates and polar coordinates. **(3.E)** Graph polar equations by plotting points and using technology.
(4) Number and Measure The student uses process standards in mathematics to apply **appropriate techniques**, **tools**, and **formulas** to **calculate measures** in mathematical and real-world problems.	**(4.I)** Use vectors to model situations involving magnitude and direction. **(4.J)** Represent the addition of vectors and the multiplication of a vector by a scalar geometrically and symbolically. **(4.K)** Apply vector addition and multiplication of a vector by a scalar in mathematical and real-world problems.

Focus on Disciplinary Literacy	Mathematical Process Standard (F) – Analyze mathematical relationships to connect and communicate mathematical ideas.
	Mathematical Process Standard (G) – Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

LEARNING SUPPORTS BY LESSON

There is a checkmark for the math support if the lesson	Lessons → Math Supports	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections	✓			✓	✓	✓		✓	✓			
makes use of graphic organizers	Graphic Organizers			✓		✓	✓				✓		
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives				✓		✓	✓	✓	✓			
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids	✓	✓		✓	✓		✓		✓	✓		✓
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports	✓	✓	✓	✓	✓	✓				✓		
includes strategies that support language development													
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓	✓					✓		✓	✓	✓
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share				✓	✓	✓	✓		✓	✓	✓	
includes sentence stems to support students with explanations	- Sentence Stems												
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics	✓		✓			✓						
includes websites or equipment that enhances the lesson	Technological Support				✓		✓	✓	✓	✓			✓
content can be presented in different forms													
uses hands-on tools or manipulatives to represent the math	- Concrete												
uses drawings to represent the math	- Pictorial	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

ROADMAP

AT A GLANCE: Unit 6 - Analytic Geometry: Vectors, Parametric Equations, and Polar Curves			
Day	Date	Lesson	Lesson Title
1		1	Introduction to Two-Dimensional Vectors
2		2	Vector Operations
3		3	Resolving Vectors and Unit Vectors
4		4	The Dot Product
5		5	Angle Between Two Vectors
6			*Unit 6 Success Day Alpha – Review Vectors*
7		6	Introduction to Parametric Equations
8		7	Converting Between Parametric and Cartesian Forms
9		8	Modeling Linear Motion with Parametric Equations
10		9	Modeling Projectile Motion with Parametric Equations
11			*Unit 6 Success Day Beta – Review Parametric Equations*
12		10	Polar Coordinate System and Graphing Polar Coordinates
13		11	Polar Coordinate Conversion
14		12	Polar Equation Conversion
15			*Unit 6 Success Day Gamma – Review Polar Coordinates*
16			*Unit 6 Success Day Delta – Assessment Review*
17			End of Unit 6 Assessment

Date: ________

Lesson 1: Introduction to Two-Dimensional Vectors

Standard(s)

◆ **(4.I)** Use vectors to model situations involving magnitude and direction.

Important Vocabulary

- **Vector**
- **Magnitude**
- **Scalar**
- **Equivalent Vectors**
- **Opposite Vectors**
- **Terminal/Initial Point**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (8 min)
- INM (25 min)
- Student Practice (15 min)
- Debrief (2 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

This lesson introduces vectors as objects that contain magnitude and direction. This lesson activates prior knowledge of right triangles which lead to the concept of a vector and its components. By the end of the lesson, describe parts of a vector, use the HMT rule to write vectors in $\langle x, y \rangle$ notation, and compute the magnitude of a vector.

Opportunities to CFU

- ✓ What are the components to a vector?
- ✓ Where on the xy-plane may a vector have its initial point?
- ✓ What does it mean for two vectors to be equivalent?
- ✓ What does it mean for two vectors to be opposite?

Other Notes to Inform Your Planning

- This lesson introduces the basics of vectors, their components, and notation. In Lessons 2-5, we extend the notation of a vector to working with linear combinations of vectors, the dot product, and determine the angle between two vectors.
- In AP Calculus BC, students revisit vectors and calculate derivatives of vector-valued functions as well as determining a particular solution given a rate vector via integration.

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2. For time $t \geq 0$, a particle moves in the xy-plane with position $(x(t), y(t))$ and velocity vector $\left\langle (t-1)e^{t^2}, \sin\left(t^{1.25}\right) \right\rangle$. At time $t = 0$, the position of the particle is $(-2, 5)$.

    (a) Find the speed of the particle at time $t = 1.2$. Find the acceleration vector of the particle at time $t = 1.2$.

    (b) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 1.2$.

    (c) Find the coordinates of the point at which the particle is farthest to the left for $t \geq 0$. Explain why there is no point at which the particle is farthest to the right for $t \geq 0$.

Lesson Look Fors

Look for teachers to...

- ❑ Show different representations of vector notation.
- ❑ Engages students in a discussion on similarities and differences from #3 in the INM.
- ❑ Engages students in a discussion on opposite vectors from #4 in the INM.

Look for students to...

- ❑ Drawing and defining vectors using appropriate notation.
- ❑ Calculate magnitude using the distance formula.

Student Know/Do Chart

- Know: A vector is an object that contains both magnitude and direction.
- Know: Vectors can be expressed algebraically in their component form. The magnitude is the length (or size) of a vector.
- Know: Two vectors with the same component form are considered equivalent.
- Do: Describe characteristics of vectors and compute magnitude.

Date: ________

Lesson 2: Vector Operations

Standard(s)

◆ **(4.J)** Represent the addition of vectors and the multiplication of a vector by a scalar geometrically and symbolically.

Important Vocabulary

- **Scalar**
- **Vector Addition**
- **Scalar Multiplication**
- **Parallelogram Method**
- **Triangle Method**

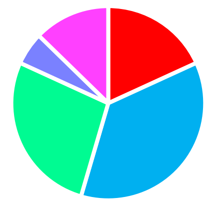
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (10 min)
- INM (20 min)
- Student Practice (15 min)
- Debrief (3 min)
- Exit Ticket (7 min)



Mathematical Goal of this Lesson

In this lesson, students will develop formulas to perform vector addition and scalar multiplication and will graph their geometrical representations on the xy-plane.

The Do Now engages students to find the resultant vector by using both the Triangle and Parallelogram Method.

In the INM, students will have the opportunity summarize their observations from the Do Now by writing the formulas for vector addition and scalar multiplication of a vector.

This lesson will also include how to find the magnitudes of vectors after addition and multiplication by a scalar has been performed.

Opportunities to CFU

- ✓ What are the two methods in combining two vectors?
- ✓ What affects does multiplying by a scalar have on a vector?

Other Notes to Inform Your Planning

- This lesson introduces the basics of vectors, their components, and notation. In Lessons 3-5, we extend the notation of a vector to working with linear combinations of vectors, the dot product, and determine the angle between two vectors.
- In AP Calculus BC, students revisit vectors and calculate derivatives of vector-valued functions as well as determining a particular solution given a rate vector via integration.

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2. For time $t \geq 0$, a particle moves in the xy-plane with position $(x(t), y(t))$ and velocity vector $\left\langle (t-1)e^{t^2}, \sin\left(t^{1.25}\right)\right\rangle$. At time $t = 0$, the position of the particle is $(-2, 5)$.

   (a) Find the speed of the particle at time $t = 1.2$. Find the acceleration vector of the particle at time $t = 1.2$.

   (b) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 1.2$.

   (c) Find the coordinates of the point at which the particle is farthest to the left for $t \geq 0$. Explain why there is no point at which the particle is farthest to the right for $t \geq 0$.

Lesson Look Fors

Look for teachers to…

- ❑ Show vector addition using the triangle and parallelogram methods.

Look for students to…

- ❑ Represent vectors algebraically and geometrically.
- ❑ Compute linear combinations of vectors and their magnitudes.

Student Know/Do Chart

- Know: Summing the horizontal and vertical components of vectors $\vec{u}$ and $\vec{v}$ produces the resultant vector $\vec{u} + \vec{v}$.
- Know: Multiplying a vector $\vec{u}$ by a scalar $c = 0$ will stretch or compress the vector. Depending on the value of c, it may change its direction as well.
- Do: Perform vector addition.
- Do: Perform scalar multiplication on vectors.

Date: ________

Lesson 3: Resolving Vectors and Unit Vectors

Standard(s)

(4.I) Use vectors to model situations involving magnitude and direction.

Important Vocabulary

- **Unit Vector**
- $\hat{\imath}, \hat{\jmath}$
- **Direction Angle**
- **Resolving a Vector**

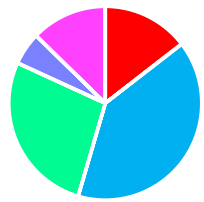
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (8 min)
- INM (22 min)
- Student Practice (15 min)
- Debrief (3 min)
- Exit Ticket (7 min)



Mathematical Goal of this Lesson

In this lesson, students will be discovering how to revolve a vector given its magnitude the unit vector, learning how to rewrite any given vector as a linear combination of the unit vectors $\vec{\imath}$ and $\vec{\jmath}$, find the direction angles of vectors, and finally, resolve vectors into components. By the end of the lesson, students will find unit vectors, write vectors as a linear combination of unit vectors, compute direction angles, and resolve vectors into components.

Opportunities to CFU

- ✓ What trig function relates to the adj and hyp of a triangle? Opp and hyp??
- ✓ How do you solve for θ given a trigonometric ratio?

Other Notes to Inform Your Planning

- This lesson introduces the basics of vectors, their components, and notation. In Lessons 4-5, we extend the notation of a vector to working with linear combinations of vectors, the dot product, and determine the angle between two vectors.
- In AP Calculus BC, students revisit vectors and calculate derivatives of vector-valued functions as well as determining a particular solution given a rate vector via integration.

2. For time $t \geq 0$, a particle moves in the xy-plane with position $(x(t), y(t))$ and velocity vector $\left\langle (t-1)e^{t^2}, \sin\left(t^{1.25}\right)\right\rangle$. At time $t = 0$, the position of the particle is $(-2, 5)$.

(a) Find the speed of the particle at time $t = 1.2$. Find the acceleration vector of the particle at time $t = 1.2$.

(b) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 1.2$.

(c) Find the coordinates of the point at which the particle is farthest to the left for $t \geq 0$. Explain why there is no point at which the particle is farthest to the right for $t \geq 0$.

Lesson Look Fors

Look for teachers to…

- ❑ Activates students' prior knowledge of vector components, direction, and magnitude.
- ❑ Represents vector components as unit vectors.

Look for students to…

- ❑ Finding components of a vector given magnitude and direction.

Student Know/Do Chart

- Know: The direction angle of a vector in standard form will be the angle θ that the vector makes with the positive x-axis.
- Know: The horizontal and vertical components of any given vector can be described by $\|\vec{v}\|\cos\theta$ and $\|\vec{v}\|\sin\theta$, respectively.
- Know: A unit vector is a vector that has a magnitude of 1.
- Do: Resolve vectors.
- Do: Write vectors as linear combinations of $\vec{\mathbf{i}}$ and $\vec{\mathbf{j}}$.

Date: ________

Lesson 4: The Dot Product

Standard(s)

- **(4.I)** Use vectors to model situations involving magnitude and direction.

Important Vocabulary

- Dot Product (Inner Product)

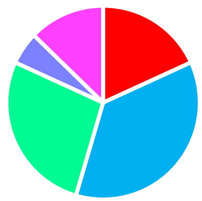
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- GeoGebra Activity
- Document camera

Lesson Structure:

- Do Now (10 min)
- INM (20 min)
- Student Practice (15 min)
- Debrief (3 min)
- Exit Ticket (7 min)



Focus on Disciplinary Literacy

INM

Mathematical Goal of this Lesson

In this lesson, we will build the intuition of the dot product and how it related to common direction of two vectors. The dot product is useful when calculating angles between vectors (as seen in the next lesson) as well as solving problem involving physics (later lesson).

Opportunities to CFU

- ✓ What does a dot product represent?
- ✓ When is the dot product at its optimum? When does it equal 0?

Other Notes to Inform Your Planning

- This lesson introduces the basics of vectors, their components, and notation. In Lessons 4-5, we extend the notation of a vector to working with linear combinations of vectors, the dot product, and determine the angle between two vectors.
- In AP Calculus BC, students revisit vectors and calculate derivatives of vector-valued functions as well as determining a particular solution given a rate vector via integration.

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2. For time $t \geq 0$, a particle moves in the xy-plane with position $(x(t), y(t))$ and velocity vector $\left\langle (t-1)e^{t^2}, \sin\left(t^{1.25}\right)\right\rangle$. At time $t = 0$, the position of the particle is $(-2, 5)$.

   (a) Find the speed of the particle at time $t = 1.2$. Find the acceleration vector of the particle at time $t = 1.2$.

   (b) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 1.2$.

   (c) Find the coordinates of the point at which the particle is farthest to the left for $t \geq 0$. Explain why there is no point at which the particle is farthest to the right for $t \geq 0$.

Lesson Look Fors

Look for teachers to…

- ❑ Uses the Mario Kart® analogy for obtaining optimal boost.
- ❑ Shows the GeoGebra activity that models the dot product as common direction.

Look for students to…

- ❑ Discuss how a driver in Mario Kart® gets the best boost.
- ❑ Identifies how dot product behaves for different pairs of vectors with common initial points.

Student Know/Do Chart

- **Know:** The dot product represents the common direction between two vectors. Positive values indicate two vectors have at least some common direction, 0 means no common direction, and negative values indicate that vectors are opposite in direction.
- **Know:** The dot product of a vector to itself is the square of the magnitude of that vector.
- **Do:** Compute dot products of two vectors.

Date: ________

Lesson 5: Angle Between Two Vectors

Standard(s)

- **(4.I)** Use vectors to model situations involving magnitude and direction.

Important Vocabulary

- **Angle Between Two Vectors**
- **Perpendicular (Orthogonal)**

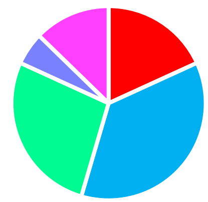
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (10 min)
- INM (20 min)
- Student Practice (15 min)
- Debrief (3 min)
- Exit Ticket (7 min)



Focus on Disciplinary Literacy

INM

Mathematical Goal of this Lesson

In this lesson, we will explore the concept of the dot product and its connection between the common direction between two vectors. The dot product is useful when calculating angles between vectors as well as solving problem involving physics (later lesson). By the end of the lesson, students will compute the angle between two vectors and determine if two vectors are orthogonal.

Opportunities to CFU

- ✓ How can the dot product be used to determine the angle between two vectors?
- ✓ What does it mean for a dot product of two vectors to be 0?

Other Notes to Inform Your Planning

- This lesson finalizes all the topics for vectors.
- In AP Calculus BC, students revisit vectors and calculate derivatives of vector-valued functions as well as determining a particular solution given a rate vector via integration.

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2. For time $t \geq 0$, a particle moves in the xy-plane with position $(x(t), y(t))$ and velocity vector $\left\langle (t-1)e^{t^2}, \sin\left(t^{1.25}\right)\right\rangle$. At time $t = 0$, the position of the particle is $(-2, 5)$.

(a) Find the speed of the particle at time $t = 1.2$. Find the acceleration vector of the particle at time $t = 1.2$.

(b) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 1.2$.

(c) Find the coordinates of the point at which the particle is farthest to the left for $t \geq 0$. Explain why there is no point at which the particle is farthest to the right for $t \geq 0$.

Lesson Look Fors

Look for teachers to…

- ❑ Uses the Law of Cosines to model two vectors with common initial points.
- ❑ Proves using the Law of Cosines the formula for the angle between two vectors.

Look for students to…

- ❑ Apply the formula to find angle between two vectors.
- ❑ Identify when two vectors are perpendicular.

Student Know/Do Chart

- Know: The dot product quantifies common direction between two vectors and is used to calculate the angle between two vectors.
- Know: The dot product of orthogonal vectors is always 0, that is, both vectors have no common direction.
- Do: Calculate the angle between two vectors.
- Do: Determine when two vectors are orthogonal.

Date: ________

Lesson 6: Introduction to Parametric Equations

Standard(s)

◆ **(3.A)** Graph a set of parametric equations.

Important Vocabulary

- **Parametric Equations**
- **Parameters**
- Plane Curve

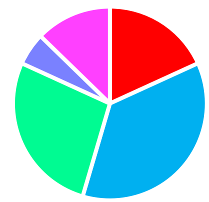
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Colored pens/pencils
- Document camera

Lesson Structure:

- Do Now (10 min)
- INM (20 min)
- Student Practice (15 min)
- Debrief (3 min)
- Exit Ticket (7 min)



Focus on Disciplinary Literacy

INM

Mathematical Goal of this Lesson

In this lesson, students will be introduced to a new system called **parametric equations**. Parametric equations are explicit functions of t that help model particle motion, projectile motion, and more. In this course, students will only be familiarizing with parametric equations for x and y.

By the end of the lesson, students will generate graphs of parametric equations over a given interval, with and without technology.

Opportunities to CFU

- ✓ What is the independent and dependent variables in a set of parametric equations?
- ✓ What are some similarities between a parametric curve and its Cartesian curve?

Other Notes to Inform Your Planning

- This topic is only 4 days. The first two days allow for students to get acquainted with implicitly defined equations and how parametric curves are graphed. Lesson 8 and 9 will go over basic applications of parametric equations.
- In AP Calculus BC, students will apply the knowledge and skills in Pre-Calculus to solve this free-response problem from the 2012 AP Calculus AB released exam. In this problem, students are investigating the particles movement defined by $(x(t), y(t))$ using its derivatives.

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

Lesson Look Fors

Look for teachers to…

- ❑ Differentiates or facilitates a discussion on how parametric curves differ from Cartesian curves.
- ❑ Incorporates technology as a tool to verify student work and further analyze curves.

Look for students to…

- ❑ Plots points $(x(t), y(t))$ on the xy-plane given a set of parametric equations.

Student Know/Do Chart

- Know: Parametric equations include a third parameter, t, when discussing how x and y values are changing over time.
- Know: A parametric equation has only one independent variable t that both x and y depend on, $(x(t), y(t))$, while Cartesian equations are usually a function of x, $f(x)$.
- Know: Parametric curves can be plotting on the xy-plane.
- Do: Graph parametric curves on the xy-plane.

Date: ________

Lesson 7: Converting Between Parametric and Cartesian Forms

Standard(s)

(3.B) Convert parametric equations into rectangular relations and convert rectangular relations into parametric equations

Important Vocabulary

- Parametric Equations
- Parameters
- Plane Curve

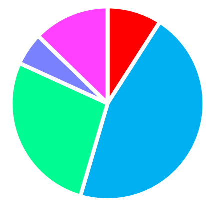
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos Activity
- Document camera

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Student Practice (15 min)
- Debrief (3 min)
- Exit Ticket (7 min)



Focus on Disciplinary Literacy

INM

Mathematical Goal of this Lesson

This lesson will extend on the concept of parametric equations from graphical to an analytical standpoint. We will explore the relationship between parametric and Cartesian forms through conversions. Since a parametric curve is modeled as $x(t)$ and $y(t)$ through t, it is possible to find an equation that produces the same shape in Cartesian form ($f(x), f(y)$, or $f(x, y)$). By the end of the lesson, students will convert between parametric equations and rectangular equations.

Opportunities to CFU

- ✓ What do parametric curves tell us that Cartesian curves don't?
- ✓ How does the 1-1 property assist in solving logarithmic equations? (INM)

Other Notes to Inform Your Planning

- This lesson will heavily depend on algebra since converting parametric equations to Cartesian requires substitution and elimination. Consider reviewing this skill as a Do Now replacement. Teacher discretion is advised.
- In AP Calculus BC, students will apply the knowledge and skills in Pre-Calculus to solve this free-response problem from the 2012 AP Calculus AB released exam. In this problem, students are investigating the particles movement defined by $(x(t), y(t))$ using its derivatives.

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

Lesson Look Fors

Look for teachers to...

- ❑ Uses the Desmos link to model curves in both parametric and Cartesian forms.
- ❑ Activates prior knowledge in solving for a variable via elimination.

Look for students to...

- ❑ Selects an appropriate variable to eliminate t when converting to Cartesian forms.
- ❑ Uses tech to verify student work.

Student Know/Do Chart

- **Know:** Parametric equations include a third parameter, t, when discussing how x and y values are changing over time.
- **Know:** A parametric equation has only one independent variable t that both x and y depend on, $(x(t), y(t))$, while Cartesian equations are usually a function of x, $f(x)$.
- **Know:** Parametric curves can be converted and represented as a relation $f(x, y)$.
- **Do:** Convert between parametric and Cartesian forms.

Date: ________

Lesson 8: Modeling Linear Motion with Parametric Equations

Standard(s)

- **(3.C)** Use parametric equations to model and solve mathematical and real-world problems.

Important Vocabulary

- Parametric Equations
- Parameters
- Plane Curve
- **Linear Motion**

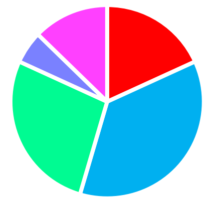
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos Activity
- Document camera

Lesson Structure:

- Do Now (10 min)
- INM (20 min)
- Student Practice (15 min)
- Debrief (3 min)
- Exit Ticket (7 min)



Mathematical Goal of this Lesson

In this lesson, we will explore an application of parametric equations called linear motion. The goal of this lesson is to model straight line motion along the x and y directions of an object as well as constructing the parametric model. By the end of the lesson, students will use parametric equations to model real-world scenarios involving linear motion.

Opportunities to CFU

✓ What is the rate of change of an object traveling along a line?

Other Notes to Inform Your Planning

- Lesson 8 and 9 are applications of parametric equations. Since objects in Lesson 8 are traveling at constant rates, students will use algebra to determine velocities. Otherwise, calculus would be needed. See below.
- In AP Calculus BC, students will apply the knowledge and skills in Pre-Calculus to solve this free-response problem from the 2012 AP Calculus AB released exam. In this problem, students are investigating the particles movement defined by $(x(t), y(t))$ using its derivatives.

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

Lesson Look Fors

Look for teachers to…

- ❑ Uses the Do Now Desmos link to activate an investigative activity.
- ❑ Give pause time when asking probing questions about the Desmos activity.

Look for students to…

- ❑ Model linear behavior along the horizontal and vertical directions using average rates of change (INM).
- ❑ Articulating the formula of linear motion.

Student Know/Do Chart

- Know: Parametric equations can be used to model motion.
- Know: Parametric equations can tell us more about the trajectory of a particle that curve on the xy-plane can't.
- Do: Create models for particle traveling along a line.
- Do: Use parametric equations to answer questions about a particle traveling along a line.

Date: ________

Lesson 9: Modeling Projectile Motion with Parametric Equations

Standard(s)

- **(3.C)** Use parametric equations to model and solve mathematical and real-world problems.

Important Vocabulary

- Parametric Equations
- Parameters
- Plane Curve
- **Projectile Motion**

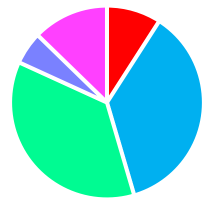
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos Activity
- Document camera

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Student Practice (20 min)
- Debrief (3 min)
- Exit Ticket (7 min)



Focus on Disciplinary Literacy

INM

Mathematical Goal of this Lesson

In this lesson, we will extend the applications of parametric equations from linear motion to projectile motion where position models follow a nonlinear trajectory.

In the Do Now, students engage in the understanding how a projectile launched from an initial position will follow a parabolic trajectory in the Cartesian plane. In the INM, students will apply the concept from the Do Now to a concrete example and will engage in a discussion of how angles and initial speed of a projectile can be modeled using a set of parametric equations, ignoring other factors like air resistance.

Opportunities to CFU

- ✓ What type of curve can model projectiles in motion?
- ✓ How do vectors help us determine velocities along the axes?

Other Notes to Inform Your Planning

- While motion is not constant in this lesson, projectiles in motion can be easily modeled, derived from calculus, using a quadratic equation, ignoring air resistance.
- In AP Calculus BC, students will apply the knowledge and skills in Pre-Calculus to solve this free-response problem from the 2012 AP Calculus AB released exam. In this problem, students are investigating the particles movement defined by $(x(t), y(t))$ using its derivatives.

For $t \ge 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

Lesson Look Fors

Look for teachers to…

- ❑ In the Do Now, the teacher asks probing questions on how the ball's trajectory will look.
- ❑ Unpacks the formulas for projectile motion.
- ❑ Uses the Desmos Activity to show a projectile in motion in real time.

Look for students to…

- ❑ Discuss the trajectory of projectiles in motion.

Student Know/Do Chart

- Know: Parametric equations can be used to model nonlinear motion.
- Know: Projectile motion will always be quadratic.
- Do: Create models for a projectile motion.
- Do: Use parametric equations to answer questions about a projectile motion.

Date: ________

Lesson 10: Polar Coordinate System and Graphing Polar Coordinates

Standard(s)

◆ **(3.D)** Graph points in the polar coordinate system and convert between rectangular coordinates and polar coordinates.

Important Vocabulary

- Rectangular (Cartesian) Coordinate System
- **Polar Coordinate Plane**
- **Polar Coordinate System**
- **Polar Symmetry**
- **Polar Coordinates**

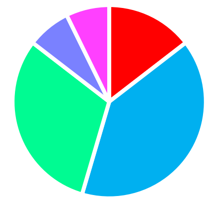
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Rulers
- Document camera
- Protractors

Lesson Structure:

- Do Now (8 min)
- INM (22 min)
- Student Practice (17 min)
- Debrief (4 min)
- Exit Ticket (4 min)



Mathematical Goal of this Lesson

Throughout this unit students will be introduced to the Polar Coordinate system and how it relates to the Cartesian coordinate system.

This first lesson will focus on introducing the polar coordinate system and plotting polar coordinates. The INM will build on student's previous knowledge of the development and geometry of reference angles and their relation to the Unit circle. Students will utilize the skills of developing coterminal angles as previously covered in Unit 4.

Students will master the day's objective by completing problems to develop their fluency with polar and rectangular plotting.

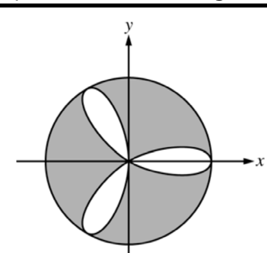
Opportunities to CFU

✓ What is the relationship between Cartesian coordinate points and polar coordinate points?

Focus on Disciplinary Literacy
INM

Other Notes to Inform Your Planning

- In AP Calculus AB/BC, students will extend their knowledge of polar coordinates and polar equations to calculate derivatives of functions written in polar coordinates and calculate areas of regions defined by polar curves using definite integration.



91. The figure above shows the graphs of the polar curves $r = 2\cos(3\theta)$ and $r = 2$. What is the sum of the areas of the shaded regions?

(A) 0.858 (B) 3.142 (C) 8.566 (D) 9.425 (E) 15.708

Lesson Look Fors

Look for teachers to…

- ❑ Demonstrates the connection between a Cartesian coordinate and polar.

Look for students to…

- ❑ Plotting polar points and equivalent points.

Student Know/Do Chart

Know: Cartesian coordinate system is based on two directional lengths while the polar coordinate system represents an angle measure.

Know: Relationship to convert between polar and Cartesian coordinate systems can be developed using right triangle geometry and the basic trigonometric identities.

Do: Graph and find additional representations of polar coordinates.

Date: ________

Lesson 11: Polar Coordinate Conversion

Standard(s)

- **(3.E)** Graph points in the polar coordinate system and convert between rectangular coordinates and points.

Important Vocabulary

- **Polar Coordinate Conversion**
  - $x = r\cos\theta$
  - $y = r\sin\theta$
  - $r^2 = x^2 + y^2$
- $\tan\theta = \frac{y}{x}$

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Ruler
- Document camera
- Protractor

Lesson Structure:

- Do Now (8 min)
- INM (25 min)
- Student Practice (14 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In the first lesson of the unit, students are introduced to plotting a polar coordinate on a polar coordinate grid. Students have had experience plotting equivalent coterminal polar coordinate points.

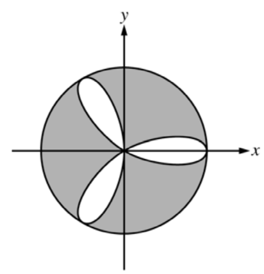
This lesson will focus on converting between polar and rectangular (Cartesian) coordinate systems. The INM will develop student's previous knowledge of right triangle geometry and polar systems, so they may determine the equations to convert between the two coordinate systems. Students will validify the relationship between the coordinate systems by verifying their findings through ruler measurements.

Opportunities to CFU

- ✓ How is direction calculated?
- ✓ How do you calculate r?

Other Notes to Inform Your Planning

- In AP Calculus AB/BC, students will extend their knowledge of polar coordinates and polar equations to calculate derivatives of functions written in polar coordinates and calculate areas of regions defined by polar curves using definite integration.



91. The figure above shows the graphs of the polar curves $r = 2\cos(3\theta)$ and $r = 2$. What is the sum of the areas of the shaded regions?

(A) 0.858 (B) 3.142 (C) 8.566 (D) 9.425 (E) 15.708

Lesson Look Fors

Look for teachers to…

- ❏ Explains how a Cartesian Coordinate and its corresponding Polar Coordinate are similar.

Look for students to…

- ❏ Uses $\cos\theta = \frac{x}{r}$ when finding x-coordinate of a coordinate pair.
- ❏ Uses $\sin\theta = \frac{y}{r}$ when finding y-coordinate of a coordinate pair.

Student Know/Do Chart

- Know: The Cartesian coordinate system is based on two directional lengths while the polar coordinate system represents an angle measure.
- Know: Relationship to convert between polar and Cartesian coordinate systems can be developed using right triangle geometry and the basic trigonometric identities.
- Do: Convert between basic polar equations and rectangular equations.

Date: ________

Lesson 12: Polar Equation Conversion

Standard(s)

(3.E) Graph points in the polar coordinate system and convert between rectangular coordinates and points.

Important Vocabulary

- Polar Coordinate Conversion
  - $x = r\cos\theta$
  - $y = r\sin\theta$
  - $r^2 = x^2 + y^2$
- $\tan\theta = \frac{y}{x}$

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Unit Circle
- Document camera

Lesson Structure:

- Do Now (5 min)
- INM (30 min)
- Student Practice (12 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

The previous two lessons of the unit introduced students to plotting a polar coordinate on a polar coordinate grid. Students also developed the relationship between rectangular and polar coordinate systems.

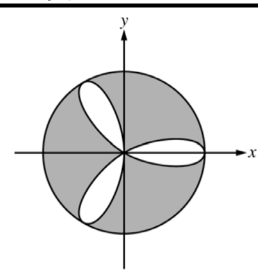
This lesson will further develop the relationship between the two systems. Students will convert between polar equations and rectangular (Cartesian) equations. The INM will develop the student's understanding of the relationship between the polar and rectangular systems by examining the similarities of the graphs written in each system.

Opportunities to CFU

✓ What are some properties of trigonometry we can use to help convert between polar and Cartesian forms?

Other Notes to Inform Your Planning

- In AP Calculus AB/BC, students will extend their knowledge of polar coordinates and polar equations to calculate derivatives of functions written in polar coordinates and calculate areas of regions defined by polar curves using definite integration.



91. The figure above shows the graphs of the polar curves $r = 2\cos(3\theta)$ and $r = 2$. What is the sum of the areas of the shaded regions?

(A) 0.858 (B) 3.142 (C) 8.566 (D) 9.425 (E) 15.708

Lesson Look Fors

Look for teachers to…

- ❑ Find opportunities to show conversions done in Desmos.

Look for students to…

- ❑ Use $r^2 = x^2 + y^2$ when converting between forms.
- ❑ Use $\tan\theta = \frac{y}{x}$ when converting between forms.

Student Know/Do Chart

- **Know:** The Cartesian coordinate system is based on two directional lengths while the polar coordinate system represents an angle measure.
- **Know:** Relationship to convert between polar and Cartesian coordinate systems can be developed using right triangle geometry and the basic trigonometric identities.
- **Do:** Convert between basic polar equations and rectangular equations.

Recommended Unit 6 Success Day Material and Resources

Date: __________

To review **topics based on your data on Success Days**, use the following resources. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer aligned practice and teacher-led groups.

To review or practice vectors:
Khan Academy Vectors Unit
Content Video Lessons:

- Introduction to Vectors and Scalars
- Recognizing Vectors
- Equivalent Vectors
- Vector Magnitude from Graph
- Vector Magnitude from Initial and Terminal Points
- Adding and Subtracting Vectors
- Graphically Adding and Subtracting Vectors
- Graphically Subtracting Vectors
- Adding Vectors Algebraically & Graphically
- Scalar Multiplication of Vectors
- Analyzing Scalar Multiplication
- Combined Vector Operations

To review or practice parametric equations:
Content Video Lessons:

- Sketching a Parametric Curve
- Parametric Equations for Linear Motion
- Parametric Equations for Projectile Motion

To review or practice polar coordinates or polar equations:
Content Video Lessons:

- Expressing a Polar Point Multiple Ways
- Converting Coordinates
- Converting Between Polar and Rectangular Equations, Ex 1
- Converting Between Polar and Rectangular Equations, Ex 2
- Converting Between Polar and Rectangular Equations, Ex 3
- Graphing Polar Curves – Part 1
- Graphing Polar Curves – Part 2

UNPACKED STANDARDS

Focus standards for this unit.

<table>
<tr><th colspan="3">Standards Clarification</th></tr>
<tr><th>Standards</th><th>Specificity</th><th>Notes/Explanations/Examples</th></tr>
<tr>
<td>(4.K) Apply <u>vector addition and multiplication of a vector by a scalar</u> in mathematical and real-world problems.</td>
<td><u>Concepts:</u>
Vectors are objects that can be characterized by
• direction, and
• magnitude
Vector Addition
Let $\boldsymbol{v} = \langle v_1, v_2 \rangle$ and $\boldsymbol{u} = \langle u_1, u_2 \rangle$ be vectors. Then, the sum of $\boldsymbol{v}$ and $\boldsymbol{u}$ is
$\langle v_1 + u_1, v_2 + u_2 \rangle$.
Vector Multiplication by a Scalar
Let $\boldsymbol{v} = \langle v_1, v_2 \rangle$ and c be a real-valued scalar. Then,
$c\boldsymbol{v} = \langle cv_1, cv_2 \rangle$.</td>
<td>2010 SCORING GUIDELINES (Form B)

Question 2

The velocity vector of a particle moving in the plane has components given by
$\frac{dx}{dt} = 14\cos(t^2)\sin(e^t)$ and $\frac{dy}{dt} = 1 + 2\sin(t^2)$, for $0 \le t \le 1.5$.
At time $t = 0$, the position of the particle is $(-2, 3)$.
(a) For $0 < t < 1.5$, find all values of t at which the line tangent to the path of the particle is vertical.
(b) Write an equation for the line tangent to the path of the particle at $t = 1$.
(c) Find the speed of the particle at $t = 1$.
(d) Find the acceleration vector of the particle at $t = 1$.

(a) The tangent line is vertical when $x'(t) = 0$ and $y'(t) \ne 0$.
On $0 < t < 1.5$, this happens at $t = 1.253$ and $t = 1.144$ or 1.145.
2 : { 1 : sets $\frac{dx}{dy} = 0$; 1 : answer }

(b) $\left.\frac{dy}{dx}\right|_{t=1} = \frac{y'(1)}{x'(1)} = 0.863447$
$x(1) = -2 + \int_0^1 x'(t)\,dt = 9.314695$
$y(1) = 3 + \int_0^1 y'(t)\,dt = 4.620537$
The line tangent to the path of the particle at $t = 1$ has equation $y = 4.621 + 0.863(x - 9.315)$.
4 : { 1 : $\left.\frac{dy}{dx}\right|_{t=1}$; 1 : $x(1)$; 1 : $y(1)$; 1 : equation }

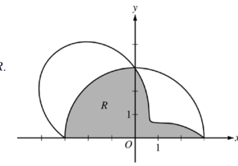
(c) Speed $= \sqrt{(x'(1))^2 + (y'(1))^2} = 4.105$
1 : answer

(d) Acceleration vector: $\langle x''(1), y''(1) \rangle = \langle -28.425, 2.161 \rangle$
2 : { 1 : $x''(1)$; 1 : $y''(1)$ }</td>
</tr>
</table>

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**
(3.C) Use parametric equations to model and solve mathematical and real-world problems.	**Concepts:** **Parametric Equations** • Parametric equations are a set of equations that express a set of quantities as explicit functions of a number of independent variables, known as "parameters." • $x(t)$ • $y(t)$ • t (independent variable)	**2008 SCORING GUIDELINES (Form B)** **Question 1** A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = \sqrt{3t}$ and $\frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right)$. The particle is at position $(1, 5)$ at time $t = 4$. (a) Find the acceleration vector at time $t = 4$. (b) Find the y-coordinate of the position of the particle at time $t = 0$. (c) On the interval $0 \leq t \leq 4$, at what time does the speed of the particle first reach 3.5 ? (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 4$. (a) $a(4) = \langle x''(4), y''(4)\rangle = \langle 0.433, -11.872\rangle$ — 1 : answer (b) $y(0) = 5 + \int_4^0 3\cos\left(\frac{t^2}{2}\right) dt = 1.600$ or 1.601 — 3 : { 1 : integrand; 1 : uses $y(4) = 5$; 1 : answer } (c) Speed $= \sqrt{(x'(t))^2 + (y'(t))^2}$ $= \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)} = 3.5$ The particle first reaches this speed when $t = 2.225$ or 2.226. — 3 : { 1 : expression for speed; 1 : equation; 1 : answer } (d) $\int_0^4 \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)}\, dt = 13.182$ — 2 : { 1 : integral; 1 : answer }

Standards Clarification

Standards	Specificity	Notes/Explanations/Examples

(3.D) Graph points in the polar coordinate system and convert between rectangular coordinates and polar coordinates.	**Concepts:**
	Polar coordinate system
	• Continuity on a closed interval
	• Continuity at a point
	• Types of discontinuities
	  ○ Removable discontinuities
	  ○ Nonremovable discontinuities
	Rectangular coordinate system
	• If f is a continuous function on the closed interval $[a, b]$ and d is a number between $f(a)$ and $f(b)$, then the Intermediate Value Theorem guarantees that there is at least one number c between a and b such that $f(c) = d$.
	Content:
	Including, but not limited to:
	• Assessing the continuity of a function…
	  ○ at a point
	  ○ on a closed interval
	  ○ endpoints on a closed interval
	• Identifying and verifying the hypotheses for If, Then statements
	• Identifying the conclusion to an If, Then statement given the hypotheses are satisfied.
		AP Calculus AB 2014 Released FRQ #2
		AP® CALCULUS BC
		2014 SCORING GUIDELINES
		Question 2
		The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \le \theta \le \pi$.
		
		(a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.
		(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.
		(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.
		(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \ge 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.
		———
		(a) Area $= \frac{9\pi}{4} + \frac{1}{2}\int_0^{\pi/2} (3 - 2\sin(2\theta))^2 \, d\theta = 9.708$ (or 9.707) — 3 : { 1 : integrand; 1 : limits; 1 : answer }
		(b) $x = (3 - 2\sin(2\theta))\cos\theta$; $\left.\frac{dx}{d\theta}\right\|_{\theta=\pi/6} = -2.366$ — 2 : { 1 : expression for x; 1 : answer }
		(c) The distance between the two curves is $D = 3 - (3 - 2\sin(2\theta)) = 2\sin(2\theta)$. $\left.\frac{dD}{d\theta}\right\|_{\theta=\pi/3} = -2$ — 2 : { 1 : expression for distance; 1 : answer }
		(d) $\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = \frac{dr}{d\theta} \cdot 3$; $\left.\frac{dr}{dt}\right\|_{\theta=\pi/6} = (-2)(3) = -6$ — 2 : { 1 : chain rule with respect to t; 1 : answer }

VERTICAL STANDARDS

This section details the **progression** of key student expectations/standards** in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students** for in the subsequent course.

6th Grade	Pre-Calculus	AP Calculus AB/BC
• **(8.A)** Extend previous knowledge of triangles and their properties to include the sum of angles of a triangle, the relationship between the lengths of sides and measures of angles in a triangle, and determining when three lengths form a triangle. **Algebra 1** • **A.2A** Determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities. • **A.2B** Write linear equations in two variables in various forms, like $y = mx + b, Ax + By = C,$ and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points. • **A.2C** Write linear equations in two variables given a table of values, a graph, and a verbal description. • **A.3C** Graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.	• **(3.A)** Graph a set of parametric equations. • **(3.B)** Convert parametric equations into rectangular relations and convert rectangular relations into parametric equations. • **(3.C)** Use parametric equations to model and solve mathematical and real-world problems. • **(3.D)** Graph points in the polar coordinate system and convert between rectangular coordinates and polar coordinates. • **(3.E)** Graph polar equations by plotting points and using technology. • **(4.I)** Use vectors to model situations involving magnitude and direction. • **(4.J)** Represent the addition of vectors and the multiplication of a vector by a scalar geometrically and symbolically. • **(4.K)** Apply vector addition and multiplication of a vector by a scalar in mathematical and real-world problems.	• **CHA-3.G.1** Methods for calculating derivatives of real-valued functions can be extended to parametric functions. • **CHA-3.G.2** For a curve defined parametrically, the value of $\frac{dy}{dx}$ at a point on the curve is the slope of the line tangent to the curve at that point. $\frac{dy}{dx}$, the slope of the line tangent to a curve defined using parametric equations, can be determined by dividing $\frac{dy}{dt}$ by $\frac{dx}{dt}$, provided dt does not equal zero. • **CHA-3.6.3** $\frac{d^2y}{dx^2}$ can be calculated by dividing $\frac{d}{dt}\left(\frac{dy}{dx}\right)$ by $\frac{dx}{dt}$. • **CHA-6.B.1** The length of a parametrically defined curve can be calculated using a definite integral. • **CHA-3.H.1** Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions. • **CHA-5.D.1** The concept of calculating areas in rectangular coordinates can be extended to polar coordinates. • **CHA-5.D.2** Areas of regions bounded by polar curves can be calculated with definite integrals. • **FUN-3.G.1** Methods for calculating derivatives of real-valued functions can be extended to functions in polar coordinates. • **FUN-3.G.2** For a curve given by a polar equation $r = f(\theta)$, derivatives of $r, x,$ and y with respect to θ, and first and second derivatives of y with respect to x can provide information about the curve. • • **FUN-8.A.1** Methods for calculating integrals of real-valued functions can be extended to parametric or vector-valued functions. • **FUN-8.B.1** Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along a curve in the plane defined using parametric or vector-valued functions. • **FUN-8.B.2** For a particle in planar motion over an interval of time, the definite integral of the velocity vector represents the particle's displacement (net change in position) over the interval of time, from which we might determine its position. The definite integral of speed represents the particle's total distance traveled over the interval of time. __________ NOTE: These "essential knowledge" (EK) standards are from The College Board Concept Outline for AP Calculus AB/BC, not the TEKS.